The optimal holding period for a commercial Real Estate, Portfolio: Taking the Lease Structure into Account

Fabrice Barthélémy (CEMOTEV), Charles-Olivier Amedée-Manesme (Université de Laval), Michel Baroni (ESSEC Buisness school)

Cahier du CEMOTEV n° 2014-01
The optimal holding period for a commercial Real Estate, Portfolio: Taking the Lease Structure into Account

Fabrice Barthélémy (CEMOTEV), Charles-Olivier Amedée-Manesme (Université de Laval), Michel Baroni (ESSEC Business school)

Cahier du CEMOTEV n° 2014-01
The Optimal Holding Period for a Commercial Real Estate Portfolio: Taking the Lease Structure into Account

Charles-Olivier Amédée-Manesme
Laval University, Department of Finance, Insurance and Real Estate, Quebec, G1V 0A6, QC, Canada
charles-olivier.amedee-manesme@fsa.ulaval.ca

Fabrice Barthélémy
CEMOTEV, Université de Versailles Saint-Quentin-en-Yvelines, 78047 Guyancourt Cedex33, France
fabrice.barthelemy@uvsq.fr

Michel Baroni
ESSEC Business School, Avenue Bernard Hirsch – B.P. 105, 95021, Cergy-Pontoise Cedex, France.
baroni@essec.edu

Abstract

Purpose
The purpose of this paper is to determine an optimal holding period for a real estate portfolio when break-options are included in the leases.

Methodology/approach
Optimization is achieved by maximizing the portfolio value, which is considered to be the sum of the discounted free cash flows and the discounted terminal value. The terminal value and the market rental value are simulated by a stochastic process representing a real estate index. To model the recurrent cash-flows, the lease structure of the portfolio and the behaviour of the tenant must be taken into account. A unit of real estate is vacated by a tenant if its current rent is too high in comparison with the market rental value available for similar properties. Vacant spaces remain unused for a period of time that is estimated using Poisson’s law.

Findings
We demonstrate that consideration of the break-options included in leases modifies the optimal holding period for a real estate fund. We show how factors included in the model influence the optimal time at which to sell the portfolio.

Practical implications
Practitioners are offered a practical methodology for determining the optimal best holding period for a portfolio or for a property.

Originality/value
The originality of the paper derives from considering the possibility that a tenant might move, which would modify the cash flows. This is critical in real estate portfolio management, because such specific risk is difficult to diversify.

Keywords: Real estate, Portfolio management, Simulation, Optimal time to sell
I. Introduction

The optimal holding period in real estate portfolio management is an issue that has only recently drawn the attention of investors and academics. Institutional investors have only recently realized the importance of the holding period for managing the risk of real estate assets or, more generally, real estate portfolios. For many years, real estate portfolio investment was a passive process, with investors purchasing real estate, a mature asset that generated recurring cash flows (rent). The traditional strategy was to hold real estate for many years, a valid strategy given the large transaction costs and limited liquidity of real estate investments. Few institutional investors bought opportunistic or value-added assets held for shorter periods. This latter type of asset was the preserve of developers, some opportunistic funds and specialized REITs. Given the specialization and sophistication of real estate practice and of the industry in general, investors now examine the holding period of a real estate asset or portfolio as a parameter in considering when or whether to invest. However, the holding period continues to be treated as just one parameter, dependent upon either the lease structure or length life of a fund, rather than upon a result computed from expectations of the performance of the economy as a whole or of the real estate market (e.g., initial yields, future developments and the development of rental values). Underlying this reasoning is the criterion of liquidity, a key issue for investors. An asset leased for a long period is more liquid than one leased for a short period, even in a bull market. Many investors look for long-term leases without break-options, because they do not want to carry leasing risk. Depending on the local market, they look for long-term leases (e.g., 10-year leases) and sell the asset after a few years (e.g., 5 years) to an investor interested in the opportunities asset management may offer, including the renegotiation and marketing of assets. They thus adapt the holding period strategy to the lease structure.

Investors adjust real estate holding-period strategies in to the lease structure. This can be explained by the various kinds of investors in the real estate industry. Often for purely technical reasons, a finite holding period (conventionally 10 years in corporate finance) is used in cash flow projections to avoid an infinitely long (>30 years) cash flow
series. The choice of holding period in cash flow projections must fit either the exit strategy or investors’ objectives intentions (some close-ended funds have finite lives and constrain the initial strategy). If the predefined holding period is short, the weight of terminal value in net present value is more important, and the expected risk reflected by this value is also more important. From our viewpoint, investors decide to sell a property for three main reasons:

- The property had been managed intensively and no further asset management is planned. Asset managers often argue that their work consists of managing a property by securing tenants (lease and relation) and doing work on the property, such as a change in property use, energy consumption, security, parking, cleaning, etc.
- The asset belongs to a portfolio for which an exit strategy was defined initially. Depending on the portfolio management strategy chosen by the investor, the policy regarding resale may be different. Core investors that were once interested only in long-term leases without break-options sold all properties when the lease lengths were below a given duration (e.g., 5 years). An opportunistic investor who seeks large capital returns might be interested only in properties that require significant asset management, such as repositioning or refurbishment. This investor sells properties as soon as these processes are complete.
- The asset does not fit the portfolio sufficiently well. This might be the case when too many leasing risks are concentrated into the same period, or when market rental values are far lower or higher than the current rent.

All of these cases have one thing in common: the property is sold when the period remaining before expiry of the lease is sufficiently long to make the property attractive and liquid to the buyer. Selling an obsolete, vacant property is never the objective in a business plan that optimizes portfolio value. An investor has to take this opportunity, which occurs during the holding period. The buyer must bear vacancy and redevelopment risks, and thus might offer a price that incorporates this risk (such as a large discount).
Our objective is to consider risk linked to lease structure in future cash flows, and to determine how taking the lease structure of a property into account modifies the optimal holding of a real estate portfolio. Given that the behaviour of a landlord with respect to the holding period is determined primarily by the lease structure, we model changes in holding period caused by the specific lease structure. This is achieved through a combination of Monte Carlo simulations for the estimation of terminal values and market rental values, and option theory to simulate the exercise of break-options.

One of the contributions made by this article is that it takes account of idiosyncratic risk. This is fundamental, because real estate portfolios require a very large number of assets in order to diversify specific risk (cf. Brown, 1998; Byrne and Lee, 2001; Callender et al., 2007). Generally, investors do not hold sufficient assets to adequately diversify specific risk. Therefore, the consideration of specific risk improves real estate portfolio management, and in particular impacts on the determination of the holding period. If the market is correctly replicated through the portfolio, the optimal holding period can be computed using market data alone. If not (generally the case in real estate), the optimal holding period must take account of specific risk in the portfolio, here including leasing risk as a fundamental risk. This is exactly what we consider in the present paper.

The structure of the paper is as follows. The following section surveys the relevant literature and presents existing models. First of all, the literature on holding periods is presented. Secondly, we emphasize that the classical DCF method does not enable the optimal holding period to be computed. Thirdly, we present results from existing research (Baroni et al. (2007b)), in which the terminal value is treated as a diffusion process. This model provides a formula that can accurately determine the optimal holding period for a real estate portfolio. Fourthly, we present the model used to determine the optimal holding period, taking into account the timing of leases. In the final section, we apply the proposed model, and show how the optimal holding period responds to changes in various parameters and lease structures.
II. The Literature and existing models

A. The Literature

The real estate holding period has long been an issue both for those who study real estate and those who actually work in the real markets. The holding period is an essential element of investment in commercial real estate portfolios. However, calculations of the optimal holding period are nearly always empirical and the period is assumed to depend upon many factors, including market conditions, transactions costs, types of property, lease length, and strategy.

The holding period is also a classic issue in finance and has been the subject of many studies. For a long time, the literature has focussed mainly on stocks. Demsetz (1968) and Tinic (1972) noticed that transaction costs influence holding periods. Amihud and Mendelson (1986) show that assets with high bid-ask spreads (a proxy for high transaction costs) are kept at equilibrium by investors who expect to hold assets for a long time. In an empirical study, Atkin and Dyl (1997) consider the effects of firm size, bid-ask spread and volatility of returns on holding periods of stocks from 1981 to 1993, for a sample of NASDAQ and NYSE firms. They demonstrate a positive correlation between holding period, transaction costs and firm size, and a negative one between holding period and price variability. Two assertions are generally accepted in the stock literature, namely that large transaction costs cause investors to hold assets for a long time, and that substantial volatility causes investors to hold an asset for a shorter period. Real estate assets exhibit these two features of high transaction costs and significant volatility, which is precisely why the optimal holding period, represents a challenge both for academics and practitioners in the field.

Real estate holding periods are the subject of many empirical studies, but no consensus has emerged and the literature is not particularly extensive in terms of the range of issues covered. For the USA, Hendershott and Ling (1984) and Gau and Wang
(1994) argue holding durations are conditioned by tax laws. For the UK, the relationship between returns and holding periods appears to be complex. In a study based on interviews with investors, Rowley et al. (1998) show that investors and new property developers have a specific holding period in mind from the start. They conclude that, for office space, a holding period decision is linked to depreciation and obsolescence. For retail property, the decision is more empirical, depending on active property management and the prevailing situation in the commercial property market. In a more recent article, Collett et al. (2003) highlight the fact that fixation of a holding period by the investors is important for any decision to invest in commercial real estate portfolios. Investment appraisal requires a specified analysis period, and asset allocation depends on the variances and covariances of assets influenced by a reference interval. Using the UK database of properties provided by IPD over an 18-year period, they observe that the median holding period is about seven years. Sales rates vary across the holding period (probably due to rent cycles and lease structures), and the holding period varies by property type. The larger and more expensive the properties, the longer the holding period. If the return is greater, the holding period is shorter. However, even if Collett et al. (op. cit.) suggest a link between price volatility and holding period, they fail to highlight a proxy for measuring the relationship. For small residential investments, Brown and Geurts (2005) offer an empirical response to the following questions: how long does an investor own an apartment building, and why do investors sell some properties more frequently than do others? Using a sample of apartment buildings of between 5 and 20 units over the period 1970 to 1990 in San Diego, California, they found the average holding period to be approximately five years. They conclude that investors sell property earlier, when values rise faster than rent. Using a microeconomic framework, Brown (2004) shows that consideration of risk that is specific to real estate investments explains why private investors actually own real estate, and also their buying and selling behaviour, which is more driven by entrepreneurial decision criteria than by financial ones used for other assets. Consistent with this conclusion, applying the CAPM for individuals as a way of understanding portfolio management does not lead to relevant results, as demonstrated by Geltner et al. (2006). However, for residential real estate, Cheng et al. (2010) demonstrate that higher illiquidity and transaction costs lead to longer
holding periods, while higher return volatility implies shorter holding periods. These latter results are consistent with previous articles on financial assets. Taking a different approach, Baroni et al. (2007a) set out to determine the optimal holding period, using dynamic cash flows for rent inflows and terminal values in real estate portfolio management. These dynamics are considered as simple diffusion processes in which the corresponding parameters are, respectively, rental and price trends and volatility. The parameters have been estimated from a rent index and a real estate index using French data for Paris, considering the correlation between these two indices. This approach suggests that the role played by the holding duration in determining asset value is significant. Baroni et al. (2007b) determine the optimal holding period ex ante (for example for closed funds, when the initial investment is realized). They model terminal values as diffusion processes, and derive a closed formula for the optimal holding period, given a number of parameters. Barthélémy and Prigent (2009) also compute an optimal time to sell a diversified portfolio in three cases, assuming the investor knows the following: the probability distribution of the real estate index, each possible path of the market dynamics, and at any time, the maximum value he/or she can expect from the portfolio. Finally, Barthélémy and Prigent (2011) considered the issue of the holding period in real estate from the perspective of risk aversion.

To the best of our knowledge, the existing academic literature on holding periods does not consider lease structure to be an essential factor in decision-making. However, many investors do in fact select a strategy as a function of the lease, and not only of the market or the state of the economy. We therefore consider how the lease structure determines the optimal holding period for a fund. The next sections review extant models on real estate portfolio holding periods, after which we present a model for determining the optimal time to sell if the lease durations are considered.
B. Optimal holding period with traditional discounted cash flow (DCF)

Most investors originally used the Discounted Cash Flow (DCF) framework to evaluate portfolios. It is easy to demonstrate that this framework is inappropriate for computing an optimal holding period for a portfolio of real estate assets.

The traditional and deterministic DCF model calculates net present value as the sum of all future cash flows generated by the asset, discounted by a discount rate. Let us denote $V_{0,T}$ the net present value of the asset or portfolio sold at date $T$.

$$V_{0,T} = \sum_{t=1}^{T} \frac{FCF_t}{(1+k)^t} + \frac{P_T}{(1+k)^T}$$

where $k$ is usually the weighted average cost of capital (WACC) used to discount the various free cash flows $FCF_t$, and $P_T$ is the terminal value computed as:

$$P_T = \frac{FCF_T}{k - g_\infty} (1+g_\infty)$$

where the free cash flow (FCF) after time $T$ grows infinitely at the constant rate $g_\infty$. If we denote $g$ as the growth rate of the free cash flows before time $T$, the equation becomes:

$$V_{0,T} = \sum_{t=1}^{T} \frac{FCF_t (1+g)^{t-1}}{(1+k)^t} + \frac{FCF_T (1+g)^{T-1}(1+g_\infty)}{(k - g_\infty)(1+k)^T}$$

Baroni et al. (2007b) demonstrate that the pricing behaviour can be studied by computing $V_{0,T+1} - V_{0,T}$.

$$V_{0,T+1} - V_{0,T} = FCF_1 \frac{(1+g)^{T-1}}{(1+k)^T} \left( \frac{g-g_\infty}{k-g_\infty} \right)$$

As $k > g_\infty$, the sign on the right of the equation corresponds to the sign of $g - g_\infty$. We then have the following states:

If $g > g_\infty$ then $V_{0,T+1} - V_{0,T} > 0$ and the price grows infinitely;
If \( g = g_o \) then \( V_{o,T+1} - V_{o,T} = 0 \) and the price remains stable;

If \( g < g_o \) then \( V_{o,T+1} - V_{o,T} < 0 \) and the price decreases infinitely.

Consequently, the traditional DCF framework does not enable an optimal holding period for a portfolio to be determined according to the asset present value, whatever the rates of expected growth. Given this issue, Baroni et al. (2007b) developed a model that leads to a closed formula.

C. Optimal holding period incorporating risk in the terminal value (Baroni, Barthélémy and Mokrane, 2007b)

Baroni et al. (2007a) propose a Monte Carlo simulation of valuation, and their contribution is to model terminal value. They consider that the real estate price of the assets follows a geometric Brownian motion (versus an infinite growth rate with traditional DCF):

\[
\frac{dP_t}{P_t} = \mu_p dt + \sigma_p dW_t
\]

This equation assumes that real estate returns can be modelled as a simple diffusion process, where parameters \( \mu_p \) and \( \sigma_p \) are the trend and volatility. They propose this model to improve the DCF method, and to allow for an optimal holding period. They then compare this new approach with the discrete case derived in the previous section.

Following Baroni et al. (2007a), the expected present value of the asset or portfolio sold at date \( T \) is:

\[
E(V_{o,T}) = \sum_{i=1}^{T} \frac{FCF_i}{(1+k)^i} + \frac{P_T}{(1+k)^T}
\]

with \( P_T \) computed with the Brownian process \((\mu_p, \sigma_p)\) and
\[ E(P_t) = P_0 (1 + \mu)^T \]

The expected growth rate of the present value is:

\[ E(V_{0,T+1} - V_{0,T}) = \frac{1}{(1+k)^{T-1}} \left[ FCF_t (1+g)^T + P_0 (1+\mu)^T (\mu-k) \right] \]

They conclude that two cases be considered:

- \( \mu = k \), hence there is no optimal holding period;
- \( \mu \neq k \), an optimal selling date (under existing conditions) may exist and is obtained by a closed formula\(^1\):

\[
T' = \frac{\ln \left( \frac{FCF_t}{V_0 (k-\mu)} \right)}{\ln \left( \frac{1+\mu}{1+g} \right)}
\]

This formula determines the conditions under which an optimal solution exists. The conditions can be summarized by:

\[
\max \left( g, k - \frac{FCF_t}{P_0} \right) < \mu < k
\]

An optimal holding period for a real estate portfolio can be thus derived.

For example if \( k = 8.40\% \), \( g = 3\% \), \( \mu_p = 4.5\% \), \( \sigma_p = 5\% \), \( P_0 = 100 \) and \( FCF_t = 4.8 \), an optimal holding period of about 14 years is derived (here, the free cash flow periodicity corresponds to one year).

\(^1\) The optimal holding period does not depend on the standard deviation parameter (\( \sigma_p \))
Figure 1 - Optimal holding period for the portfolio value when the terminal value is simulated

Risk can also be incorporated similarly in the market rental value. And to simulate the risk of vacancy in the cash flows, the lease structure must be taken into consideration. This is presented in the next section.

D. The break-option: optimal holding period incorporating risk in terminal value and lease structure (Amédée-Manesme, Baroni, Barthélémy and Dupuy, 2013)

For core investors, real estate assets are cash-flow-generating assets and properties purchased primarily for the rent they generate. One of the most essential determinants of cash flow in real estate investment (particularly commercial real estate) is the lease that stipulates the conditions under which the property is let. Regulating the rights and obligations of landlord and tenant, the lease provides information concerning expected cash flows for many years ahead. A lease specifies a starting date, the initial rent, lease expiration date, indexation rules, and options available to the tenant to leave
the premises before the lease expires. These last options are called break (sometimes renewal) options. They are a unique feature of continental Europe leases, and one of the most important risks real estate investors face in Europe. Lease structures vary from country to country, and are especially different in the U.K. Information on lease structure is thus an essential component of any cash flow model. A property may be vacant (sometimes partially) and may generate more costs than revenue. Vacancy is an essential issue for real estate investment, particularly for cash flow forecasting. The departure (or possible departure) of a tenant is an essential factor for valuing future cash flows.

A break option is an asymmetric right in favour of the tenant; at the time of a break option (usually fixed in the lease), the tenant has the right, but not the obligation, to terminate the lease. A tenant may leave at the time of a break option for a number of reasons. Therefore, break options bring about vacancies and hiatus in cash flows, and this represents the principal cash flow risk borne by investors. A vacant space incurs costs and yields no revenue. In addition, the premises may become run down or unfashionable or more quickly obsolete, thus also involving an increased risk of capital loss. Previous academic studies of rental contracts such as Miceli and Sirmans (1999) suggest that landlords attempt to minimize vacancy and turnover costs by offering discounts to long-term tenants. Landlords often try to dissuade a tenant from leaving at the dates fixed by the lease by offering discounts or rental holidays. In this way, they minimize the number of break options.

Risk arising from the structure of the lease is crucial when considering real estate investment, particularly in the context of an optimal holding period. A property with a short-term lease bears more leasing risk, and therefore may be less liquid. When future cash flows are projected, it is essential to consider whether a tenant may leave, and this may exert a substantial impact on terminal value and the holding period. In the following model, the impact on the holding period is considered by assuming that the terminal value is subject to systematic risk only.

2 As with any commodity or product traded in a free market, supply and demand determine rent. If a rental property is priced above the current market value, competitively priced properties are taken up quickly, while overpriced ones remain vacant. A property does not retain efficiency without a minimum level of ongoing investment. A vacant property often requires substantial investment in addition to recurrent costs (e.g., local taxes, security, technical control etc.). Vacancy cost is therefore an estimated magnitude arising as a consequence of vacancy.
Amédée-Manesme et al. (2013) develop a model that considers lease structure (and therefore the break-options) of a real estate portfolio. The objective of the authors was to improve existing commercial real estate valuation methods by introducing uncertainty and risk into the valuation process. This issue had already been discussed in French and Gabrielly (2004, 2005), Hoesli et al. (2006) and Baroni et al. (2007a), but their analysis was improved by considering the break-options. The model takes into account the exercise of such options with the induced vacancy period and considers the risk underlying the lease structure, and more precisely, the risk of the rent at the exercise date of the break-option exceeding the market rental value. Obviously, agency costs (e.g., moving costs, transaction costs etc.) must also be considered. Thus, Monte Carlo simulation and option theory are used to model a tenant’s decision, and to simulate future cash flows. The model integrates uncertainty into the determination of terminal value. Both the price of the asset (P) and market rental values (MRV) are simulated as diffusion processes:

\[
\frac{dP}{P_t} = \mu_p dt + \sigma_p dW^{P}_t
\]

\[
\frac{dMRV}{MRV_t} = \mu_{MRV} dt + \sigma_{MRV} dW^{MRV}_t
\]

These equations assume that real estate prices and rental values can be modelled as a Brownian diffusion process, where parameters \( \mu_p \) and \( \mu_{MRV} \) are the price and market rental value index trends, \( \sigma_p \) and \( \sigma_{MRV} \) the price and market rental value index volatilities. The correlation between market rental values and terminal value is also considered. Market rental value is modelled as index \( I_{MRV}^{MRV} \), but the size of the spaces and other characteristics can also be considered. Generally, two spaces located in the same property follow the same index, but differ by rent, size and specifications (e.g., floors, floors, floors, floors, floors, floors).

\(^3\) The space may be let or not let. If let, depending on the terms of the contract, the tenant enjoys the possibility of leaving at a predetermined date during the length of the lease (the break option). When the lease terminates, both tenant and landlord decide either to continue with the lease, or not to do so, possibly. The end of the lease is also a break option, but is symmetric, since both tenant and landlord can exercise it, even if doing so results in additional costs. Uncertainty regarding changes in rent over time arises from the possibility that a break option will be exercised, and from the length of vacancy periods.
A/C system, orientation etc.). However, the rent charged is not necessarily equal to the market rental value, so the model compares rent currently paid with simulated rental values. A rational tenant exercises a break-option as soon as the rent currently paid is too high in relation to the market rental values available for similar spaces. Therefore, the tenant leaves the space at the time of a break-option when rent is much higher than the market value. This is written as:

$$\frac{Rent_{t,i}}{MRV_{t,i}} > 1 + \alpha, \text{ then } Rent_{t+1,i} = 0$$

where $\alpha$ is a decision-making criterion ($\alpha \geq 0$ if the tenant is rational and includes possible moving or transaction costs, for instance), $Rent_{t,i}$ is the rent of the space $i$ at time $t$, and $MRV_{t,i}$ is the market rental value of space $i$ at time $t$.

The model considers differences that arise between the development of market rent and rent contracted into years before a break-option. Three factors are thus considered: the development of rental values (or the ways in which they are revised); the evolution of market rental values; and the evolution of possible vacancies. Indeed as soon as a tenant vacates a space, the landlord faces a void period: this length of vacancy (of the void period) is modelled using Poisson’s law.

$$X \sim P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

where $k$ is length of vacancy and $\lambda$ is a positive real number equal to the expected number of occurrences during a given interval. In this case, it is equal to average length of vacancy.

For each simulated scenario, one path of portfolio or asset price is elaborated and analysed with respect to all paths for each market rental value of each unit in the portfolio (all of them being correlated). Simulated prices are used in the discounted cash flow model as the terminal values. For each unit let and at the time of a break-option (if any), the current rent paid (passing rent) is compared with the simulated market rental value available for similar spaces. In cases where the rent paid is too high in comparison with
the market rental value, a tenant vacates the property and the landlord may face a gap in cash flows for the unit. Length of vacancy is determined randomly following Poisson’s law, which parameter equals the average vacancy length in the market. If a space is vacant initially, the length of vacancy is also determined using Poisson’s law. By assuming that both tenants and landlords act rationally, new leases are contracted at the market rental value of the unit. At each period, the price of the portfolio is calculated from the cash flows produced by the units (some cash flows are 0 for vacant spaces). The value of the portfolio can then be computed for each holding period. The procedure is replicated thousands of times to obtain a portfolio price for various holding periods. We thus obtain both the mean of all scenarios and a distribution of prices for each holding period.

For clarity of presentation and assuming rational behaviour of the players, a new lease without vacancy is assumed to be established at the end of a lease. In this sense, we assume that landlords and tenants negotiate and conclude a new lease at the MRV. This assumption can easily be relaxed and risk can also be considered at the end of the lease.

4 A space may be let or remain vacant. If let, the tenant may or may not have the option of leaving at a predetermined date during the term of the lease (the break option). When the lease terminates, both tenant and landlord can decide either to continue with the lease or not (with some limitations, which are considered irrelevant here). The end of the lease is therefore a break option, but reciprocal, since both tenant and landlord can exercise it, even if often with varying costs. Uncertainty regarding changes in rent over time arises from the possibility that the break option will be exercised, and from the length of vacancy periods.
Figure 2 - 3/6/9-year lease, indexation 2.5%/year, $I_{MRV}^{MRV} \sim N(2\%,10\%)$. The break-option is exercised and leads to a gap in cash flow.

Figure 3 - Indexation 2.5%/year, $I_{MRV}^{MRV} \sim N(2\%,10\%)$. Rent averages of 10,000 scenarios exhibit unsteady paths for a 3/6/9-year lease.

This model based on a stochastic approach, incorporates both systematic (through simulation of price) and specific (structure of lease) risk for each cash flow and is much richer than the classical DCF method. This approach is able to take account of the tenant’s behaviour regarding break options included in a lease, and can be tailored for real-world portfolio managers managing portfolio risk. Seeing the best time to sell a
portfolio is an important part of portfolio management. We consider this issue in the next section, where we apply the model in order to determine an optimal holding period.

III. Optimal holding period: illustration

It is prohibitively complex to determine an analytical formula for the optimal holding period of a real estate portfolio when options embedded in the lease are considered. Following the reasoning of Baroni et al. (2007b) and the model of Amédée-Manesme et al. (2013), we thus consider the optimal holding period as that which maximizes the portfolio value. The asset value is simulated, incorporating systematic risk both in the terminal value and the MRV, and the specific risk included in each lease, which is represented by the possibility of vacancy.

To demonstrate the relevance of the model and changes undergone by the optimal holding period when the lease structure is examined, we use the same numerical parameters as in Baroni et al. (2007b): $P_0 = 100$, $FCF_0 = 4.8$, $\mu_P = 4.5\%$, $\sigma_P = 5\%$, $k = 8.4\%$, $g = 3\%$. To illustrate more easily how to determine an optimal holding period with this example, the portfolio is considered as a unique asset with only one lease. In addition, the lease is assumed to start at the beginning of the first period. For the base case, we add the following parameters that refer to market rental values and to the lease: $MRV_0 = 4.8$ ($= FCF_0$), $\mu_R = 3\%$ ($= g$), $\sigma_R = 0$, $\lambda = 0^5$, lease structure $= 3/6/9$ years (nine-year lease with possible break options at year 3 and 6). Note that the rental path is assumed to be exactly the same as the MRV dynamics (the rent growth rate equals the MRV trend and the volatility of market rental values is set to 0). Finally, the portfolio is considered as a whole, with no possibility of rebalancing or of arbitrage.

Using 5,000$^6$ replications, we first simulate all possible values of the portfolio for the base case. We then analyse the sensitivity of the value by changing selected parameters through five cases, in order to demonstrate their influence. These parameters

---

$^5$ Average lease length.

$^6$ This is sufficient for the estimation of expectations in our case.
are the MRV volatility ($\sigma_R$ studied in case 1), the length of vacancy ($\lambda$ studied in case 2), and the lease structure (in cases 3, 4 and 5) where the possible differences between the rents indexation ($g$) and the MRV growth are taken into account ($\mu_R$). All these cases are summarized in Table 1.

Moreover, for all cases, the rental yield ($FCF_0/P_0$), price growth and volatility ($\mu_P$ and $\sigma_P$), the decision criteria ($\alpha$) and the cost of capital (WACC) are assumed to remain constant. Their corresponding values are the following (from Baroni et al. (2007b)):

$$FCF_0/P_0 = 4.8\%, \mu_P = 4.5\%, \sigma_P = 5\%, \; MRV_0 = FCF_0 = 4.8, \; \alpha = 0, \; WACC = 8.4\%$$

Setting $\alpha$ to 0 means that the break option is exercised as soon as the rent exceeds the market rental value: $Rent_{t,i} > MRV_{t,i}$ at the time of the break-option.

At this stage, it is worth noting that the interpretation of results remains valid, even though obtained for a particular set of parameters. The analysis of various cases enables us to decide whether the variation of one parameter of interest has a positive or negative impact on the optimal time to sell ($T^*$). For each case, we compute $V(0,t)$, the discounted value of the portfolio at time 0 for a holding period of $t$ years.

<table>
<thead>
<tr>
<th>Case #</th>
<th>$\sigma_R$</th>
<th>$\lambda$</th>
<th>$g$</th>
<th>$\mu_R$</th>
<th>Lease structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base case</td>
<td>0</td>
<td>0</td>
<td>3%</td>
<td>3%</td>
<td>3/6/9</td>
</tr>
<tr>
<td>Case 1</td>
<td>Varies</td>
<td>0</td>
<td>3%</td>
<td>3%</td>
<td>3/6/9</td>
</tr>
<tr>
<td>Case 2</td>
<td>0.1%</td>
<td>Varies</td>
<td>3%</td>
<td>3%</td>
<td>3/6/9</td>
</tr>
<tr>
<td>Case 3</td>
<td>10%</td>
<td>0</td>
<td>3%</td>
<td>3%</td>
<td>Varies</td>
</tr>
<tr>
<td>Case 4</td>
<td>10%</td>
<td>0</td>
<td>2%</td>
<td>4%</td>
<td>Varies</td>
</tr>
<tr>
<td>Case 5</td>
<td>10%</td>
<td>0</td>
<td>4%</td>
<td>2%</td>
<td>Varies</td>
</tr>
</tbody>
</table>

Table 1 – Summary of all cases
**Base case:** case of reference where the rent path is exactly the same as the MRV dynamics

The base case illustrated in Figure 4 presents the case where no volatility of MRV is considered.

![Graph showing discounted value at time 0](image)

**Figure 4 - Optimal holding period without volatility of market rental values**

In this case, we obtain the same results as Baroni et al. (2007b), in which the rent follows a deterministic process. This is consistent with the assumption of no volatility for the MRV:

\[
\begin{align*}
MRV_0 &= Rent_0 \\
g &= \mu_R \\
\sigma_R &= 0
\end{align*}
\]

Combining the absence of volatility and an MRV trend equal to indexation leads to the same rent and MRV in each period. Therefore, no break-options are exercised, since the rent never exceeds the market rental value. We obtain an optimal holding period of 14 years. In fact, a model without market rent volatility is equivalent to a model in which rent is indexed and only the price is volatile. It is important to note that the absence of exercised options renders the average vacancy length meaningless for the model.
For all the following cases, the MRV volatility will be strictly positive. This allows the difference between the market rental value and the rent to be taken into account at each step of the simulation.

**Case 1: Sensitivity to rent volatility (figure 5)**

Figure 5 illustrates the sensitivity of the optimal holding period to market rent volatility, by using different values of $\sigma_R$, specifically 5%, 10% or 15%. The base case is represented as the solid line. The length of vacancy ($\lambda$) is set to 0, which enables us to determine only the effect of market rent volatility on the optimal time to sell. For instance, the dashed blue line, corresponding to a market rent volatility of 5%, leads to an optimal holding period of 12 years (instead of 14 for $\sigma_R$ equal to 0). With a market rent volatility of 15% (the dashed red line), the optimum is at year 6. In addition, all the discounted portfolio values decrease as market rent volatility increases. In this case, the optimal holding period appears as negatively correlated with market rent volatility.

![Figure 5 - Optimal holding period with market rent volatility](image_url)
The market rental value does not follow the rent, and in some paths, break-options are exercised. We thus observe a decline in portfolio value, due to the number of break-options exercised. For each option exercised, the rent is revised to a lower value and the holding period decreases. We conclude that the optimal holding period decreases when the volatility of market rental value increases.

This result corresponds with the literature on stocks and bonds, which argues that the holding period usually decreases as the volatility of returns increases.

**Case 2:** sensitivity to the length of vacancy (Figure 6)

In order to consider the effect of the vacancy length, the market rent volatility is set to 0.1%, high enough to allow the exercise of options. Note that minimum volatility is mandatory. Without volatility, no options are exercised and we are back to the deterministic case presented above.

![Figure 6 - Optimal holding period with increasing vacancy length](image-url)
In this case, when the average length of vacancy increases ($\lambda$ is expressed in years), the optimal holding period decreases. A zero average vacancy length, combined with minimal market rent volatility, yields almost the same optimal holding period (the discounted portfolio values are almost the same – see the solid black line and the dashed blue line). This can be explained by the fact that low rent volatility brings the rental value close to the market rental value. An average vacancy of 6 months ($\lambda=0.5$ year), leads to $r^* = 12$ years (the green dotted line), whereas a value of 1 year or more, gives $r^* = 3$ years, the break option date. Optimal holding decreases when vacancy length increases.

This demonstrates that it is important to consider the length of vacancy, due to void periods. Sensitivity is particularly high when break-options are multiple, or when secured cash flows are of short duration. This behaviour explains why closed funds, which have contractual holding periods, seek to secure cash flows by negotiation with the tenant, proposing to exchange a lower rent for the exclusion of break options, in order to avoid vacancy.

**Case 3:** sensitivity to the lease structure when rent and MRV growth rates are equal (Figure 7)

Figure 7 shows many lease structures, from a 9-year lease contract without any break-options to a lease that can be terminated each year. We observe that a 9-year contract with the option of breaking at year 6 (blue dashed line) yields a lower optimal holding period ($r^* = 12$ years) than a 9-year contract (solid black line) without any break-options\(^7\) ($r^* = 14$ years). Adding more break options results in an even smaller holding period, as demonstrated by the green dotted line and the red dashed line.

The number of possible break options thus has a very significant impact on the optimal time to sell: the longer the length of secured cash flow, the longer the holding period. In comparison with the reference case, we observe that the more risky the lease

---

\(^7\) The absence of frictions at the end of the lease is due to the assumptions underlying the model. The new initial rent is set to the current market rental value. No vacancy length is considered. Hence, we obtain the same expected curve as the reference case.
structure, the shorter the optimal time span to selling (and the smaller the values of portfolio).

These results corroborate both the classical literature and practice with respect to stocks or bonds. The literature concludes, as do we, that more risky assets are held for a shorter period. Real estate practice also exhibits similar results: risky assets (with leasing risk) are generally bought by opportunistic investors who hold them for a shorter period of time. Our results are therefore in line with empirical observation. However this finding does not take into account the possible impact of a long lease on the asset price. In practice, a long unexpired lease may provide an incentive to sell when the market maximizes the value of such a lease.

![Figure 7 - Optimal holding period with various lease structures](image)

**Figure 7 - Optimal holding period with various lease structures**

**Case 4**: Sensitivity to the lease structure when rent increases more slowly than MRV\(^8\) (Figure 8)

\(^8\) This case happened between 2003 and 2008 in France and in many other European markets when market rental values were increasing strongly.
Figure 8 is computed with a rent growth rate of 2% and a market rental value growth rate of 4%. If current rent increases at a lower average rate than the MRV, a tenant is less tempted to leave the property, compared with the reference case. Indeed, the MRV is on average higher than the rent. Here, we present simulations conducted under the assumption of null average vacancy length, knowing that other values for $\lambda$ lead to similar results. Therefore, the few options exercised are immediately released based on a lower rent.

The dips observed in Figure 8 correspond to the end of each lease, when the tenant and the landlord readjust the rent to the rental value (this is an assumption of the model that can be relaxed).

The 9-year lease contract without any break-options (the blue solid line) exhibits two particular points at years 9 and 18. These two points correspond to the end of the leases, the one contracted at time 0 which ends at $t = 9$, and the other contracted at $t = 9$ which ends at $t = 18$. According to the model's assumption, a new lease is contracted at the current market rental value at the end of the contract. It is expected that the market rental growth will be higher than the rental growth on average, irrespective of its volatility. Thus, the initial price of the new lease will, on average, always exceed the last rental value of the previous lease structure.

The 9-year lease contract with a break-option at year 6 leads to more points at time 6, 9, 12, 15 and 18. For instance, at time 6, the option may be exercised. In this case, a new 6/9 lease is contracted until year 15, with a possibility to break it at year 12. Hence, the green line is below the blue one.

We conclude that a rent growth rate lower than the market rental value growth fosters a longer holding period, due to fewer break-options being exercised. Note that the higher the gap between these two growth rates, the greater the impact on $T^*$. 

Case 5: Sensitivity to the lease structure when MRV grows more slowly than rent (Figure 9)

If current rents increase at a higher rate than the market rental value, tenants may exercise break-options whenever it becomes possible. Hence, the optimal period until selling decreases, due to lower cash flows generated on average.

The 9-year lease contract without any break-options (the blue dashed line) displays two specific features at years 9 and 18. At these two dates, a new lease is contracted at the market rental value. It is expected that the new rent fixed at this market rental value will always be less than the previous rent.

The 9-year lease contract with a break-option at year 6 leads to more points than we obtain in Case 4. For instance, at time 6, the option is exercised quite often. Since the MRV is lower, the green curve is beneath the blue one. The more frequent the break options, the shorter the optimal holding period.

---

9 The discrepancy between the two growth rates is high and the rent volatility low.
We conclude that a rent growth rate higher than the market rental value results in a shorter holding period, due to the many break-options exercised. This case is consistent with observations in a bear market, in which tenants move more often to reduce fixed costs, including rent.

![Figure 9](image)

**Figure 9 - Optimal holding period with rent growth rate higher than the market rental value trend**

IV. Conclusion

We have shown in this article how the optimal holding period of a portfolio can be estimated by simulation, considering the specific risk cause by the lease structure. In fact, this approach is richer than a deterministic, because it allows many additional factors to be taken into account, such as rent volatility, the dynamics of the MRV, the length of possible vacancy and the dates of the break options during the lease. The combination of Monte Carlo simulation and option theory is a valid and accurate means of simulating the main risks on the cash flows, and also the systematic risk (on the terminal value and on the MRV) and the specific risk (the vacancy duration at which the rent becomes higher than the MRV and the tenant decides to leave the premises). The optimal holding period is supposed to be determined by the maximum value of the portfolio.
Using sample values to simulate the portfolio value, our main findings are that a higher volatility of the MRV shortens the optimal holding period. Similarly, the longer the duration of the average vacancy, the shorter the holding period. And finally, a large number of break options increases the risk of vacancy, and the volatility of the cash flows also reduces the optimal holding period. This risk is more acute when rent grows faster than the MRV.

All these results are consistent with the academic literature on other financial assets, but are also in accordance with the usual practices of both investors and portfolio managers. For instance, to avoid vacancy, landlords can negotiate a lower rent in exchange for foregoing the break option. This behaviour can increase the value of the portfolio and the optimal holding period.

Finally, if such simulations of cash flows are possible, they are also a pragmatic way to analyse the risk associated with all measurable factors. The model is open and all sources of risk may be introduced into the model. Similarly, the number of assets is not limited.

However if the model takes into account the correlation between the rents and the prices in a systematic approach, it does not integrate the impact of specific rental conditions on the asset price. It is obvious that such impacts may dramatically change decisions regarding the holding duration.

The decisions may also be governed by the risk aversion of the investors and the model could integrate this parameter. Future work could also improve the model, introducing the time at which it would be necessary to sell the portfolio, and the liquidity of the market.
References


